USN | | | | | | | | | | | **06MAT41**

Fourth Semester B.E. Degree Examination, June/July 2013
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Any missing data may be suitably assumed.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Employ Taylor's series method to obtain approximate value of y at x = 0.1 and x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, y(0) = 0 considering upto fourth degree term and compare the numerical solution obtained at x = 0.2 with the exact solution $y = 3(e^{2x} - e^x)$. **(06 Marks)**

b. Using Fourth order Runge-Kutta method to solve $(x+y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5, correct to 4 decimal place. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090 find y(0.4) corrected to 4 decimal places by using Milne's predictor and corrector formula (use corrector formula twice). **(07 Marks)**

2 a. Define an analytic function and obtain Cauchy-Reimann equations in the Cartesian form. **(06 Marks)**

b. Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is a harmonic function and determine the corresponding analytic function. **(07 Marks)**

c. Find the bilinear transformation that maps the points 1, i, -1 respectively onto the points i, 0, -i under the transformation find the image of $|z| < 1$. **(07 Marks)**

3 a. If f(z) = u + iv is an analytic function and f′(z) is continuous at each point with in and on a closed curve c, then show that $\int_C f(z)dz = 0$. **(06 Marks)**

b. Expand $\frac{1}{(z-1)(z-2)}$ in a region, (i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$, (iv) $0 < |z-1| < 1$, (iv) $|z-1| > 1$. **(07 Marks)**

c. Evaluate $\int_C \frac{ze^z}{(z^2-1)}dz$ where c : |z| = 2 using Cauchy's residue theorem. **(07 Marks)**

4 a. Obtain a series solution for the differential equation $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$. **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation,

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n-1)y = 0$$

leading to Legendre's polynomial. **(07 Marks)**

c. Prove the following:

$$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}, \quad J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right\}$$ **(07 Marks)**

PART – B

5 a. Fit a curve of the form $y = a + bx + cx^2$ to the data by the method of least squares. **(06 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

b. Find the lines of regression and hence find the coefficient of correlation for the following data: **(07 Marks)**

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

c. Let a and B are any two events, then prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and hence prove,
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$. **(07 Marks)**

6 a. Find the value of K, such that the following represents a finite probability distribution and find mean and standard deviation also find (i) $P(X \le 1)$, (ii) $P(X > 1)$, (iii) $P(-1 < x \le 2)$

X	-3	-2	-1	0	1	2	3
P(X)	K	2K	3K	4K	3K	2K	K

(06 Marks)

b. If 10% of the rivets produced by a machine are defective, find the probability that out of 12 randomly choosen rivets: i) Exactly 2 will be defective; ii) At least 2 will be defective; iii) None will be defective. **(07 Marks)**

c. 200 students appeared in an examination, distribution of marks is assumed to be normal with mean = $\mu = 30$ and S.D. = $\sigma = 6.25$, how many students are expected to get marks.
i) Between 20 and 40 ii) Less than 35. **(07 Marks)**

7 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis at 5% level of significance that coin is unbiased. **(06 Marks)**

b. A sample of 12 measurement of the diameter of metal ball gave the mean 7.38 mm with S.D. 1.24 mm. Find 95% and 99% confidence limits for actual diameter given $t_{0.05}(11) = 2.2$ and $t_{0.01}(11) = 3.11$. **(07 Marks)**

c. A set of similar coins are tossed 320 times and the observations are

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follows a binomial distributions. For 5df we have $x^2_{0.05} = 11.07$. **(07 Marks)**

8 a. The joint probability distribution of two discrete random variable x and y is given by the following table. Determine the marginal distribution of x and y. Also find whether x and y are independent. **(06 Marks)**

x \ y	1	3	6
1	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{18}$
3	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
6	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{36}$

b. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is an regular stochastic matrix and find the corresponding unique fixed probability vector. **(07 Marks)**

c. Explain: i) Regular and irreducible Markov chain
ii) Periodic state
iii) State distribution and higher transition probabilities. **(07 Marks)**

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